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TRANSIENT BEHAVIOR OF LUMPED-CONSTANT SYSTEMS
FOR SENSING GAS PRESSURES

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and Richard S. Cesaro

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TRANSIENT BEHAVIOR OF LUMPED-CONSTANT SYSTEMS FOR SENSING GAS PRESSURES

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SUMMARY

The development of theoretical equations describing the behavior of a lumped-constant pressure-sensing system under transient operation is presented with experimental data that show agreement with the equations. A pressure-sensing system consisting of a tube terminating in a reservoir is investigated for the transient relation between a pressure disturbance at the open end of the tube and the pressure response in the reservoir. Design parameters are presented that can be adjusted to achieve a desired performance from such a system when the system is considered as a transfer member of a control loop.

INTRODUCTION

In many control applications, the controlled variables such as fluid flow, thrust, airspeed, torque, and temperature (references 1 and 2) may be measured by pneumatic means. Such a measuring system may incorporate a tube that terminates in a reservoir having a pressure-sensitive element. An investigation concerned with the transient relation between the pressure disturbance at the mouth of such a tube and the pressure response in the reservoir as affected by changes in system dimensions and gas conditions was conducted at the NACA Lewis laboratory and is reported herein.

The transient behavior of linear systems may be described in various ways. The methods used in the present report employ: the differential equation describing the system, the transfer function (reference 3) (the Laplace transform of the response

divided by the Laplace transform of the disturbance), the indicial response (response to a unit-step disturbance), and the frequency response (steady-state response to sinusoidal inputs). In synthesizing an over-all control system, the transfer function or frequency response has greater utility; whereas, in the study of a single transfer member, the transient behavior is easily understood by the use of the indicial-response method.

Considerable research has been conducted on this problem in fields other than controls. References 4 and 5 report investigations of this problem as applied to airspeed indicators and altimeters. In these applications, the effect of the mass inertia of the flowing medium in the tube is usually so small as to be negligible; whereas, for controls a design that would permit the exclusion of the mass-inertia effect reduces the order of the equations and limits the flexibility of design. On this basis, the analyses of references 4 and 5 are not considered applicable to the study of pressure-sensing systems used in controls.

In the field of acoustics, sound transmission in pneumatic systems has been thoroughly investigated and is presented in references 6 and 7. Parts of these analyses are used in the investigation presented herein.

Experimental data in the form of indicial responses are presented to substantiate the analysis. (The symbols used are defined in appendix A).

ANALYSIS

If consideration is limited to a gas-pressure-sensing system (fig. 1) in which the volume of the reservoir is large compared with the volume of the tube, the capacitance (appendix B) may be considered a function only of the reservoir volume and the gas conditions. Also, by restriction of the analysis to the case in which the tube length is short, the dead time (length of tube divided by the speed of sound) due to transportation lag may be neglected. As a result of these two restrictions, the volume flow at any time is considered uniform along the tube length. The "inertance" and the resistance (appendix B) are therefore functions only of the tube dimensions and the gas conditions. By these restrictions, the analysis is reduced to

an investigation of a lumped-constant system (reference 8), a system that disregards the space variables and assumes disturbances to occur instantaneously throughout the system with varying magnitudes.

General Equations

Differential equation describing pressure transient. - If an adiabatic process and small pressure changes are assumed, the relation between the pressure in the reservoir and the pressure at the mouth of the tube is described by a second-order differential equation with essentially constant coefficients:

$$JC\ddot{p}_c + RC\dot{p}_c + p_c = p_d \quad (1)$$

or

$$\frac{1}{\omega_0^2} \ddot{p}_c + \frac{2\zeta}{\omega_0} \dot{p}_c + p_c = p_d \quad (2)$$

The definition and the derivation of the coefficients appear in appendix B.

Transfer function of pressure-sensing system. - By use of the Laplace transform, the transfer function for the differential equation is

$$F(s) = \frac{P_c(s)}{P_d(s)} = \frac{1}{\frac{s^2}{\omega_0^2} + \frac{2\zeta}{\omega_0} s + 1} \quad (3)$$

Indicial response. - The indicial responses in dimensionless form for various damping ratios (reference 9) are:

Case I: Underdamped, $0 \leq \zeta < 1$

$$\frac{p_c}{p_d} = 1 - \frac{e^{-\zeta \omega_0 t}}{\sqrt{1 - \zeta^2}} \cos \left(\sqrt{1 - \zeta^2} \omega_0 t - \tan^{-1} \frac{\zeta}{\sqrt{1 - \zeta^2}} \right) \quad (4)$$

Case II: Critically damped, $\zeta = 1$

$$\frac{p_c}{p_d} = 1 - e^{-\omega_0 t} - \omega_0 t e^{-\omega_0 t} \quad (5)$$

Case III: Overdamped, $\zeta > 1$

$$\frac{p_c}{p_d} = 1 - e^{-\zeta \omega_0 t} \left[\frac{\zeta}{\sqrt{\zeta^2 - 1}} \sinh \left(\sqrt{\zeta^2 - 1} \omega_0 t \right) + \cosh \left(\sqrt{\zeta^2 - 1} \omega_0 t \right) \right] \quad (6)$$

These indicial responses are plotted for various damping ratios in figure 2.

Frequency response. - The equation for the frequency response (reference 3) is

$$F(i\omega) = \frac{1}{-\left(\frac{\omega}{\omega_0}\right)^2 + 2i\zeta \frac{\omega}{\omega_0} + 1} = Ge^{i\phi} \quad (7)$$

The gain and the phase shift as functions of frequency ratio for various damping ratios are shown in figure 3.

Variation of Coefficients

In the design of a pressure-sensing system, consideration must be given to the variation of the coefficients of the differential equation describing the system. The coefficients of equation (2) vary in two distinct ways: (1) with operating pressure

and temperature (initial pressure and temperature of the transient), and (2) during a pressure transient. Because the variation is different in each case, the cases must be considered separately.

Variation of coefficients with operating level. - The undamped natural frequency is a function only of the operating temperature of the working fluid for a given system:

$$\omega_0 = \sqrt{\frac{\pi r^2}{IV}} \gamma g R_g T_0 \quad (8)$$

The percentage change in undamped natural frequency with percentage change in operating temperature is shown in figure 4.

The damping ratio varies with viscosity, temperature, and pressure of the working fluid:

$$\zeta = \frac{4\mu}{p_0 r^3} \sqrt{\frac{VLgR_g T_0}{\pi \gamma}} \quad (9)$$

Figure 5 shows the percentage change in damping ratio with percentage change in operating pressure.

Because the viscosity is a function of the gas and the temperature, a general relation showing the variation of damping ratio with temperature cannot be determined. The variation of damping ratio, however, can be shown for given gases and system dimensions. The variation of damping ratio with temperature for air in a system having dimensions such that the damping ratio is unity at standard temperature is shown in figure 6.

Variation of coefficients during pressure transient. - During a pressure transient, for adiabatic conditions, the undamped natural frequency varies with the pressure as follows:

$$\frac{\omega_0}{\omega_{0, \text{ initial}}} = \left(\frac{p_2}{p_0} \right)^{\frac{\gamma-1}{2\gamma}} \quad (10)$$

The percentage change in undamped natural frequency with a percentage change in pressure is shown in figure 7.

If a linear variation of viscosity with temperature

$$\mu = K_1 + K_2 T$$

is assumed, the change in damping ratio is a function of the change in pressure and the initial temperature, as shown by the following equation:

$$\frac{\zeta}{\zeta_{\text{initial}}} = \frac{K_1 + K_2 \left[T_0 \left(\frac{p_2}{p_0} \right)^{\frac{\gamma-1}{\gamma}} \right]}{K_1 + K_2 T_0} \left(\frac{p_2}{p_0} \right)^{\frac{\gamma-1}{2\gamma}} \quad (11)$$

The variation of damping ratio with absolute pressure for air at standard initial temperature using a linear variation of viscosity with temperature is shown in figure 8.

APPARATUS AND PROCEDURE

A schematic representation of the experimental apparatus used to obtain transient pressure data is shown in figure 9. In order to obtain an approximate step pressure disturbance at the mouth of the tube, an air stream was interrupted by a revolving slotted disk. The diameter of the disk and the speed of the motor are such that the time of travel of a point on the disk across the mouth of the tube is of the order of 1/1000 second.

The pressure response in the reservoir at the end of the tube was sensed by a commercial device for converting pressure to an electric signal. This unit, consisting of a bellows connected to a strain-gage bridge, has a natural frequency of 1100 cycles per second. The volume flow through the tube due to the movement of the bellows is negligible as compared with the volume flow due to the compressibility of the gas in the reservoir.

The bridge was energized with an audio-signal generator supplying an 8000-cycle alternating-current potential. The bridge unbalance was fed directly into a cathode-ray oscilloscope equipped with a recording camera.

The unbalance of the bridge appeared as an amplitude modulation of the carrier wave. This modulation was found to vary linearly with the pressure in the reservoir. The period of the carrier wave provided a time base.

RESULTS AND DISCUSSION

In order to determine the validity of the analysis, experimental curves of indicial responses were obtained and compared with the theoretical responses calculated from equations (4) and (6). These curves are shown in figures 10 to 12. The damping ratios and the undamped natural frequencies were varied only by changing the tube radii.

Close agreement between the theoretical and experimental responses is shown in figures 10 and 11. For these figures, the volume ratios (tube volume divided by reservoir volume) were 0.08 and 0.12, respectively.

The volume ratio was made approximately 0.50 for the data in figure 12. An examination indicates that the undamped natural frequency is lower and the damping ratio is higher than calculated. The volume ratio was made large in order to indicate the magnitude of the error involved.

If a pneumatic system is to be used as a transfer member of a control system, a specific transfer function (equation (3)), indicial response (equations (4) to (6)), or frequency response (equation (7)) will be required. In order to attain the required response, definite values of undamped natural frequency and damping ratio are specified. The dependence of the undamped natural frequency and the damping ratio on the system parameters are indicated by equations (8) and (9), respectively. From these equations, the damping ratio and the undamped natural frequency can be seen to vary with the operating pressure and the temperature; a system designed for one operating level therefore may not function properly at another. Because of these variations, the design must be such that the deviation of undamped natural frequency and damping ratio are within acceptable limits over the range of operating pressures and temperatures for which the system is designed.

For a gas at given conditions, undamped natural frequency and damping ratio are functions of the length of the tube, the radius of the tube, and the reservoir volume. If practical considerations fix one of these system dimensions, the other two are uniquely fixed for given values of frequency and damping ratio. When two of the system dimensions are fixed, the third dimension is determined by a choice of either frequency or damping ratio.

When design conditions are such that tube length and reservoir volume are at practical minimums, the undamped natural frequency can be increased further only by an increase in tube radius. This

value of tube radius may result in too low a value of damping ratio. In order to increase damping ratio to the desired value, additional resistance in the form of a restriction may be introduced into the tube. This effect is demonstrated in the example shown in figure 13. In this example, it is desired to have a damping ratio of 0.727 and an undamped natural frequency of 571 radians per second. Practical considerations limited the tube length to 16.75 inches and the reservoir volume to 0.202 cubic inch. For the desired damping ratio, the radius must be equal to 0.0215 inch. With this radius, the undamped natural frequency is 279 radians per second (fig. 13, curve a). This value is lower than the desired frequency. Inasmuch as undamped natural frequency is a function of tube length, tube radius, and reservoir volume and whereas tube length and reservoir volume are at a minimum, the undamped natural frequency can be increased to the desired value by an increase in tube radius only. This increase, however, decreases the damping ratio to 0.085 for a tube radius of 0.044 inch (fig. 13, curve b).

In order to restore the damping ratio to 0.727 and to maintain the undamped natural frequency at 571 radians per second, added resistance was introduced into the tube while maintaining the same system dimensions (fig. 13, curve c). A method for increasing resistance without changing the system dimensions is the insertion of an orifice, a wire mesh, or other restriction in the tube. Inasmuch as the system dimensions are unchanged, the resistance is increased independently of the inertance and the capacitance.

The damping ratio for figure 13, curve c, could not be calculated from the theoretical equations. It was approximated by comparing its response to curves shown in figure 2. In this way, the desired undamped natural frequency and the damping ratio were obtained, although the tube length and the reservoir volume were fixed.

SUMMARY OF RESULTS

A gas-pressure-sensing system consisting of a tube terminating in a reservoir was considered. An analysis, restricted to a system having dimensions such that it can be treated as a lumped-constant system, was presented with experimental data. The analysis assumed that the coefficients of the differential equation were constant during a pressure transient. The variation of these coefficients during a pressure transient can be calculated, however, and were presented for a particular system.

Agreement was shown between the theoretical analysis and experimental data over the limited range of conditions investigated. Because of the large number of variables, determination of the limits of application of the differential equation was impractical. It seems unlikely that large errors would be introduced, however, if the basic restrictions were adhered to; the volume of the reservoir must be large compared with the volume of the tube, the tube length must be short, and the pressure differences existing in the system must be small.

Design parameters were presented, which can be adjusted to achieve the desired transient performance from the system when it is considered as a transfer member of a control. The variations of the coefficients of the differential equation resulting from a change in the operating level of the pressure or temperature were analyzed and presented.

The method whereby the damping ratio of the system may be increased without affecting the undamped natural frequency was experimentally investigated. The method consisted in increasing the resistance to flow without changing the system dimensions. This method results in greater flexibility in design, inasmuch as a system may be designed to produce the required value of undamped natural frequency and any damping ratio less than that required.

Lewis Flight Propulsion Laboratory,
National Advisory Committee for Aeronautics,
Cleveland, Ohio, May 25, 1949.

APPENDIX A

SYMBOLS

The following symbols are used in this report:

A	area, sq ft
a	acceleration, ft/sec ²
C	capacitance, ft ⁵ /lb
c	sound velocity, ft/sec
F(s)	transfer function
F(iω)	frequency-response function
G	gain, or amplitude amplification
g	gravitational constant, ft/sec ²
i	$\sqrt{-1}$
J	inertance, lb-sec ² /ft ⁵
K ₁ , K ₂ , K ₃ , K ₄	constants
L	length of tube, ft
m	mass, slugs
P _c (s)	Laplace transform of pressure change in reservoir
P _d (s)	Laplace transform of pressure disturbance
p _c	pressure change in reservoir, p ₂ -p ₀ , lb/sq ft
p _d	pressure disturbance, p ₁ -p ₀ , lb/sq ft
p _j	pressure drop necessary to accelerate fluid in tube, lb/sq ft
p _r	pressure drop in tube due to flow resistance, lb/sq ft
p ₀	initial steady-state absolute pressure, lb/sq ft

P_1	absolute pressure at mouth of tube, lb/sq ft
P_2	absolute pressure in reservoir, lb/sq ft
Q	volume displacement in tube, cu ft
R	resistance, lb-sec/ft ⁵
R_g	gas constant, ft-lb/(lb)(°F)
r	radius of tube, ft
s	complex variable
T	absolute temperature, °R
T_0	initial steady-state absolute temperature, °R
T_2	absolute temperature in reservoir, °R
t	time, sec
V	volume of reservoir, cu ft
γ	ratio of specific heats
δ	damping constant, sec ⁻¹
ζ	damping ratio
ϕ	phase shift, radians
μ	absolute viscosity, lb-sec/sq ft
ρ	density in tube, slugs/cu ft
ρ_0	initial steady-state density in tube, slugs/cu ft
ω	input frequency, radians/sec
ω_0	undamped natural frequency, radians/sec

Subscripts:

a,b operating levels

Dots above the symbols represent derivatives with respect to time.

APPENDIX B

DERIVATION OF THEORETICAL EQUATIONS

The pressure-sensing system shown in figure 1 can be treated as a lumped-constant system and described by an ordinary differential equation using the following restrictions and method of analysis.

General Equation

Restrictions. -

(1) The tube length must be sufficiently short so that the dead time $\frac{L}{c}$ can be neglected.

(2) The dimensions of the reservoir must be such that the pressure throughout the reservoir may be considered uniform at any time.

(3) The volume of the tube must be small compared with the volume of the reservoir. On this basis, the assumption can be made that the volume flow is uniform throughout the length of the tube and is the result of the compressibility of the gas in the reservoir.

Definition of capacitance C (references 1, 6, 7, and 10). -

$$C = \frac{dQ}{dp_2}$$

The volume flow into the reservoir is equal to the decrease in volume of the gas originally in the reservoir.

$$dQ = - dV$$

therefore

$$C = - \frac{dV}{dp_2}$$

When adiabatic compression is assumed in the reservoir

$$p_2 V^\gamma = K_3$$

$$V^\gamma dp_2 + p_2 \gamma V^{\gamma-1} dV = 0$$

$$\frac{dV}{dp_2} = \frac{-V\gamma}{p_2 \gamma V^{\gamma-1}} = \frac{-V}{p_2 \gamma}$$

Therefore

$$C = \frac{V}{p_2 \gamma}$$

The value of C varies inversely with the absolute pressure in the reservoir. For small pressure changes, C is assumed to be constant.

Definition of resistance R . - When the resistance is assumed to exist only within the tube, the resistance to flow is defined as (references 1, 6, 7, and 10)

$$R = \frac{p_r}{\dot{Q}}$$

From the Hagen-Poiseuille law

$$\dot{Q} = \frac{\pi p_r r^4}{8\mu L}$$

Therefore

$$R = \frac{8\mu L}{\pi r^4}$$

The value of R varies with the absolute viscosity, which is assumed essentially constant for small changes of pressure and temperature. The Hagen-Poiseuille law is valid only for laminar flow. For small Reynolds numbers, R is constant.

Definition of inertance J . - If the motion of the fluid in the system is assumed to occur only in the tube, the inertance is defined as (references 6, 7, and 10)

$$J = \frac{p_j}{\ddot{Q}}$$

$$p_j = \frac{ma}{A}$$

$$m = \rho LA$$

The acceleration of the fluid in the tube is

$$a = \ddot{Q}$$

Then

$$p_j = \frac{\rho LA \ddot{Q}}{A^2} = \frac{\rho L}{A} \ddot{Q}$$

Therefore

$$J = \frac{\rho L}{A} = \frac{\rho L}{\pi r^2}$$

The inertance is proportional to the density. For small pressure and temperature changes, the inertance is essentially constant.

Differential equation. - The following equation can be obtained from a pressure balance across the tube:

$$p_j + p_r + p_2 = p_1$$

$$p_j + p_r + p_2 - p_0 = p_1 - p_0$$

Then

$$p_j + p_r + p_c = p_d$$

By definition

$$p_j = J \ddot{Q}$$

$$p_r = R \dot{Q}$$

$$dp_2 = \frac{dQ}{C}$$

$$\int dp_2 = \int \frac{dQ}{C}$$

$$p_2 = \frac{Q}{C} + K_4$$

when

$$p_2 = p_0$$

$$Q = 0$$

Therefore

$$p_2 - p_0 = \frac{Q}{C} = p_c$$

and

$$J\ddot{Q} + R\dot{Q} + \frac{Q}{C} = p_d$$

or

$$JC \ddot{p}_c + RC \dot{p}_c + p_c = p_d \quad (B1)$$

The general nature of this equation is so well known that a detailed discussion is unnecessary. By definition

$$\omega_0 = \frac{1}{\sqrt{JC}} \quad (\text{undamped natural frequency})$$

$$\delta = \frac{R}{2J} \quad (\text{damping constant})$$

$$\zeta = \frac{\delta}{\omega_0} = \frac{R}{2} \sqrt{\frac{C}{J}} \quad (\text{damping ratio})$$

Equation (B1) reduces to

$$\frac{1}{\omega_0^2} \ddot{p}_c + 2 \frac{\zeta}{\omega_0} \dot{p}_c + p_c = p_d \quad (B2)$$

Transfer function. - By use of the Laplace transformation, the transfer function is (references 3 and 8)

$$F(s) = \frac{P_c(s)}{P_d(s)} = \frac{1}{\frac{s^2}{\omega_0^2} + 2 \frac{\zeta}{\omega_0} s + 1} \quad (B3)$$

Indicial responses. - For a pressure disturbance p_d equal to a step input, the solutions of equation (B2) in dimensionless form are (reference 9):

Case I: Underdamped, $0 \leq \zeta < 1$:

$$\frac{p_c}{p_d} = 1 - \frac{e^{-\zeta\omega_0 t}}{\sqrt{1 - \zeta^2}} \cos \left(\sqrt{1 - \zeta^2} \omega_0 t - \tan^{-1} \frac{\zeta}{\sqrt{1 - \zeta^2}} \right) \quad (B4)$$

Case II: Critically damped, $\zeta = 1$:

$$\frac{p_c}{p_d} = 1 - e^{-\omega_0 t} - \omega_0 t e^{-\omega_0 t} \quad (B5)$$

Case III: Overdamped, $\zeta > 1$:

$$\frac{p_c}{p_d} = 1 - e^{-\zeta\omega_0 t} \left[\frac{\zeta}{\sqrt{\zeta^2 - 1}} \sinh \sqrt{\zeta^2 - 1} \omega_0 t + \cosh \sqrt{\zeta^2 - 1} \omega_0 t \right] \quad (B6)$$

Frequency response. - The frequency response is obtained from the transfer function (equation (B3)) by substituting

$$s = i\omega$$

$$F(i\omega) = \frac{1}{-\left(\frac{\omega}{\omega_0}\right)^2 + 2i\zeta \frac{\omega}{\omega_0} + 1} = Ge^{i\phi} \quad (B7)$$

Variation of Coefficients of

Differential Equation

Variation with operating level. -

1. Undamped natural frequency

$$\omega_0 = \frac{1}{\sqrt{JC}} = \sqrt{\frac{\pi r^2}{\rho_0 L} \frac{\gamma p_0}{V}}$$

$$\frac{p_0}{\rho_0} = gR_g T_0$$

$$\omega_0 = \sqrt{\frac{\pi r^2}{LV} \gamma gR_g T_0} = c \sqrt{\frac{A}{LV}} \quad (B8)$$

$$\frac{\omega_{0,a}}{\omega_{0,b}} = \sqrt{\frac{T_{0,a}}{T_{0,b}}}$$

2. Damping ratio

$$\zeta = \frac{\delta}{\omega_0} = \frac{R}{2J} \sqrt{JC} = \frac{R}{2} \sqrt{\frac{C}{J}}$$

$$\zeta = \frac{8\mu L}{2\pi r^4} \sqrt{\frac{\pi r^2 V}{\rho_0 L \gamma p_0}}$$

$$\rho_0 = \frac{p_0}{gR_g T_0}$$

$$\zeta = \frac{4\mu}{p_0 r^3} \sqrt{\frac{VL}{\pi \gamma} gR_g T_0} \quad (B9)$$

When the working fluid is assumed to be air, and the viscosity is assumed to vary linearly with temperature and is independent of pressure (reference 11)

$$\mu = 88.9 \times 10^{-9} + 0.548 \times 10^{-9} T_0$$

$$\zeta = \frac{4(88.9 + 0.548 T_0) 10^{-9}}{p_0 r^3} \sqrt{\frac{VL}{\pi \gamma} gR_g T_0}$$

For a change in operating pressure,

$$\frac{\zeta_a}{\zeta_b} = \frac{p_{0,b}}{p_{0,a}}$$

If the system dimensions are such that $\zeta = 1$ at standard temperature, the magnitude with operating temperature is

$$\zeta = (1.176 \times 10^{-4}) (88.9 + 0.548 T_0) \sqrt{T_0}$$

Variation during pressure transient. - In order to simplify the analysis, the assumption is made that only small pressure differences exist in the system at any time.

1. Undamped natural frequency

$$\rho = \frac{p_2}{g R_g T_2}$$

$$\omega_0 = \sqrt{\frac{\pi r^2}{LV} \gamma g R_g T_2}$$

$$\frac{\omega_0}{\omega_{0,\text{initial}}} = \sqrt{\frac{T_2}{T_0}}$$

If an adiabatic process is assumed during the transient

$$\frac{\omega_0}{\omega_{0,\text{initial}}} = \left(\frac{p_2}{p_0} \right)^{\frac{\gamma-1}{2\gamma}} \quad (\text{B10})$$

2. The damping ratio is

$$\zeta = \frac{4\mu}{p_2 r^3} \sqrt{\frac{VL}{\pi \gamma} g R_g T_2}$$

assuming

$$\mu = K_1 + K_2 T_2$$

$$\zeta = \frac{4(K_1 + K_2 T_2)}{p_2 r^3} \sqrt{\frac{VL}{\pi \gamma} g R_g T_2}$$

during a transient

$$\frac{\zeta}{\zeta_{\text{initial}}} = \frac{K_1 + K_2 T_2}{K_1 + K_2 T_0} \left(\frac{p_0}{p_2} \right) \left(\frac{T_2}{T_0} \right)^{\frac{1}{2}}$$

assuming adiabatic conditions during a transient

$$\frac{\zeta}{\zeta_{\text{initial}}} = \frac{K_1 + K_2 \left[T_0 \left(\frac{p_2}{p_0} \right)^{\frac{\gamma-1}{\gamma}} \right]}{K_1 + K_2 T_0} \left(\frac{p_2}{p_0} \right)^{-\left(\frac{\gamma+1}{2\gamma} \right)} \quad (\text{B11})$$

For air,

$$\frac{\zeta}{\zeta_{\text{initial}}} = \frac{88.9 + 0.548 \left[T_0 \left(\frac{p_2}{p_0} \right)^{\frac{\gamma-1}{\gamma}} \right]}{88.9 + 0.548 T_0} \left(\frac{p_2}{p_0} \right)^{-\left(\frac{\gamma+1}{2\gamma} \right)}$$

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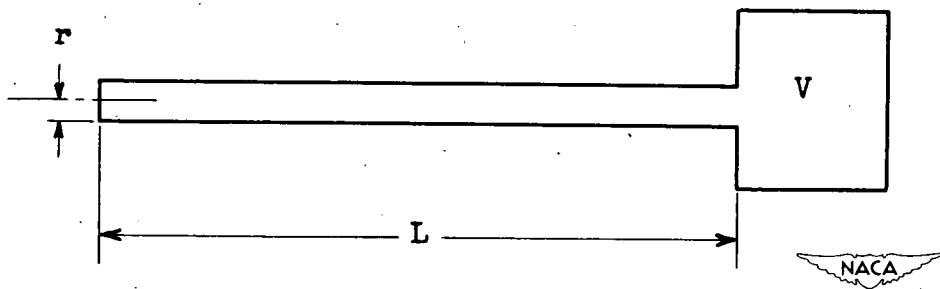


Figure 1. - Gas-pressure sensing system.

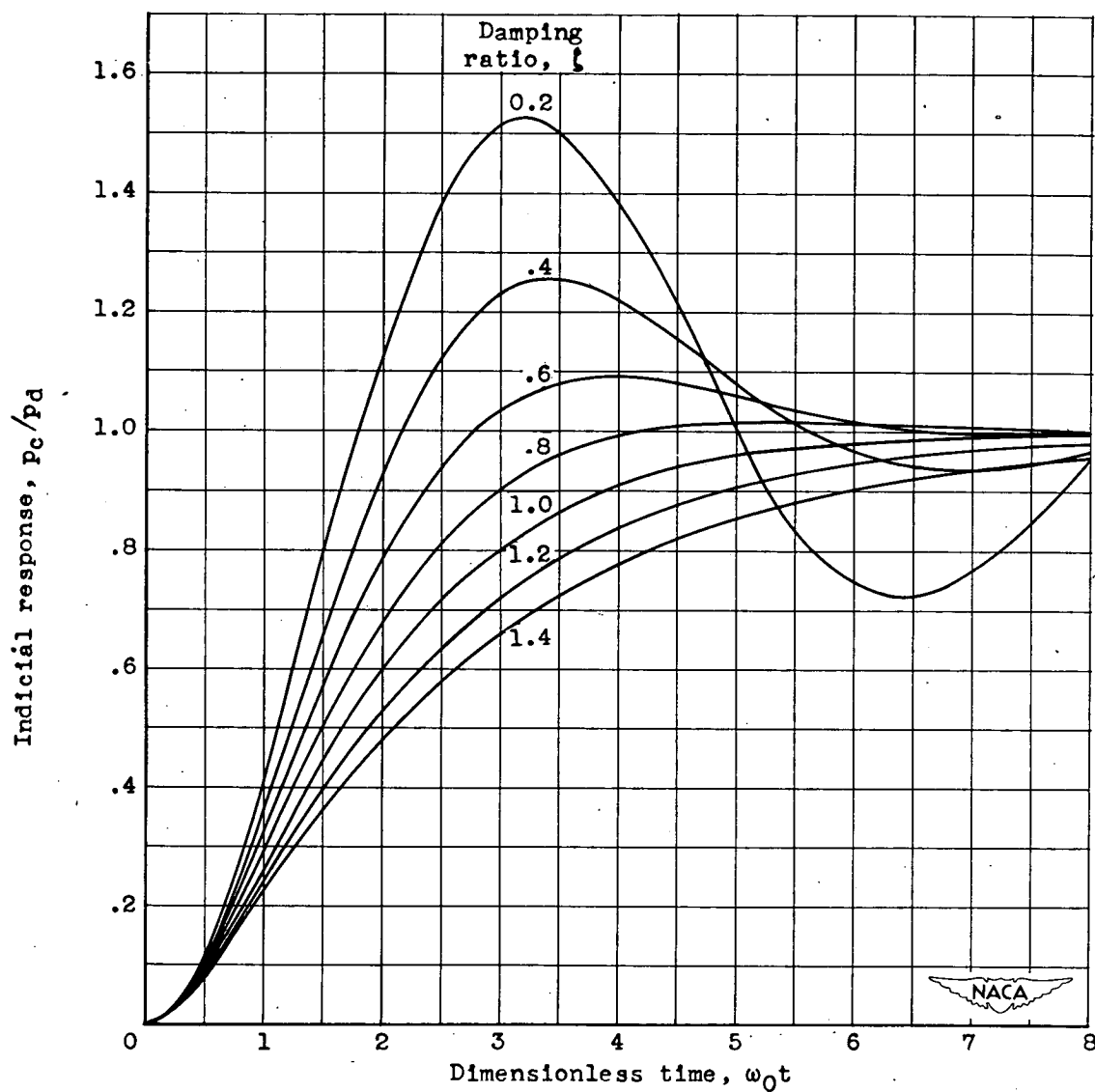


Figure 2. - Dimensionless indicial response of second-order linear differential equation with constant coefficients.

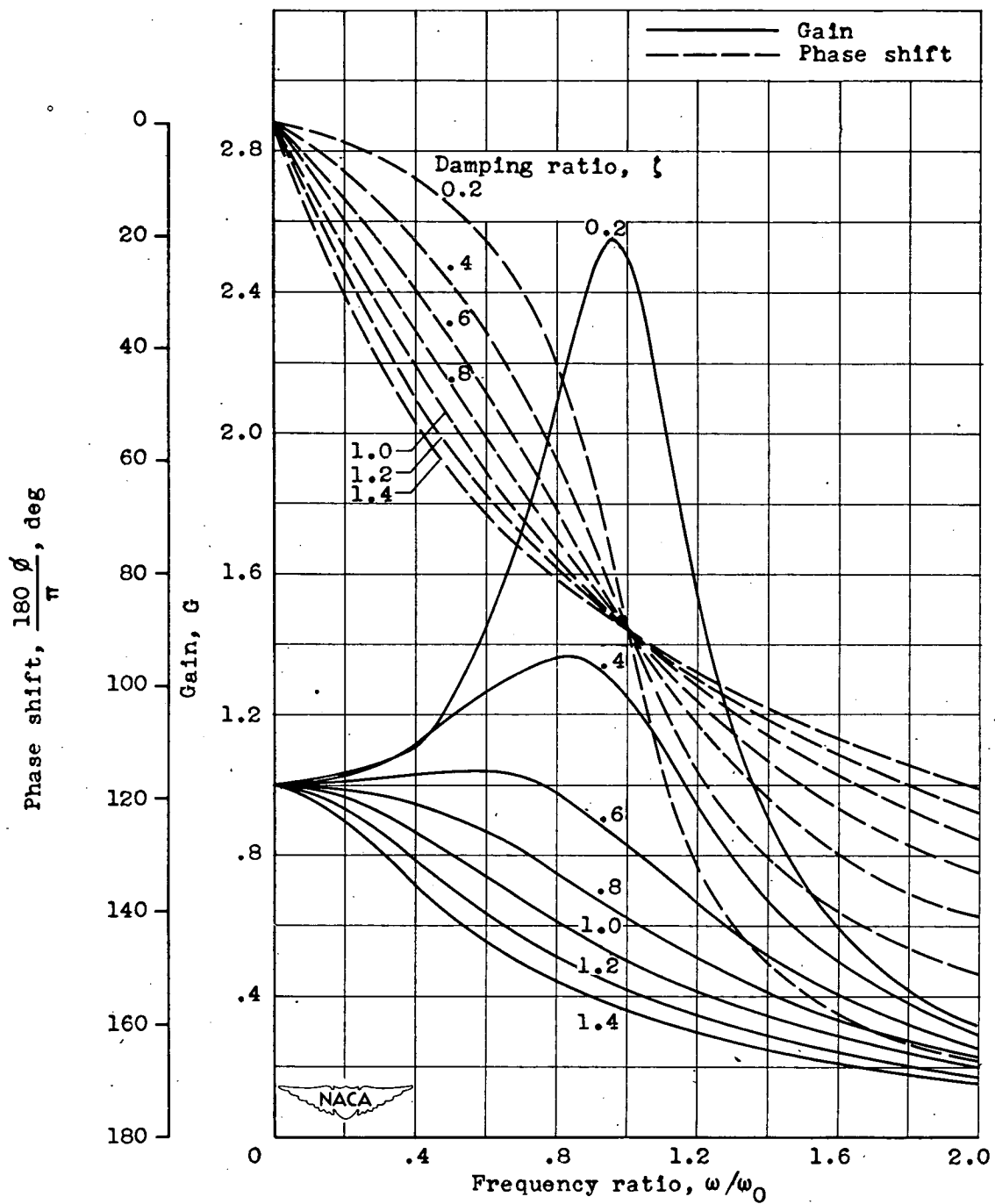


Figure 3. - Frequency response for transfer function.

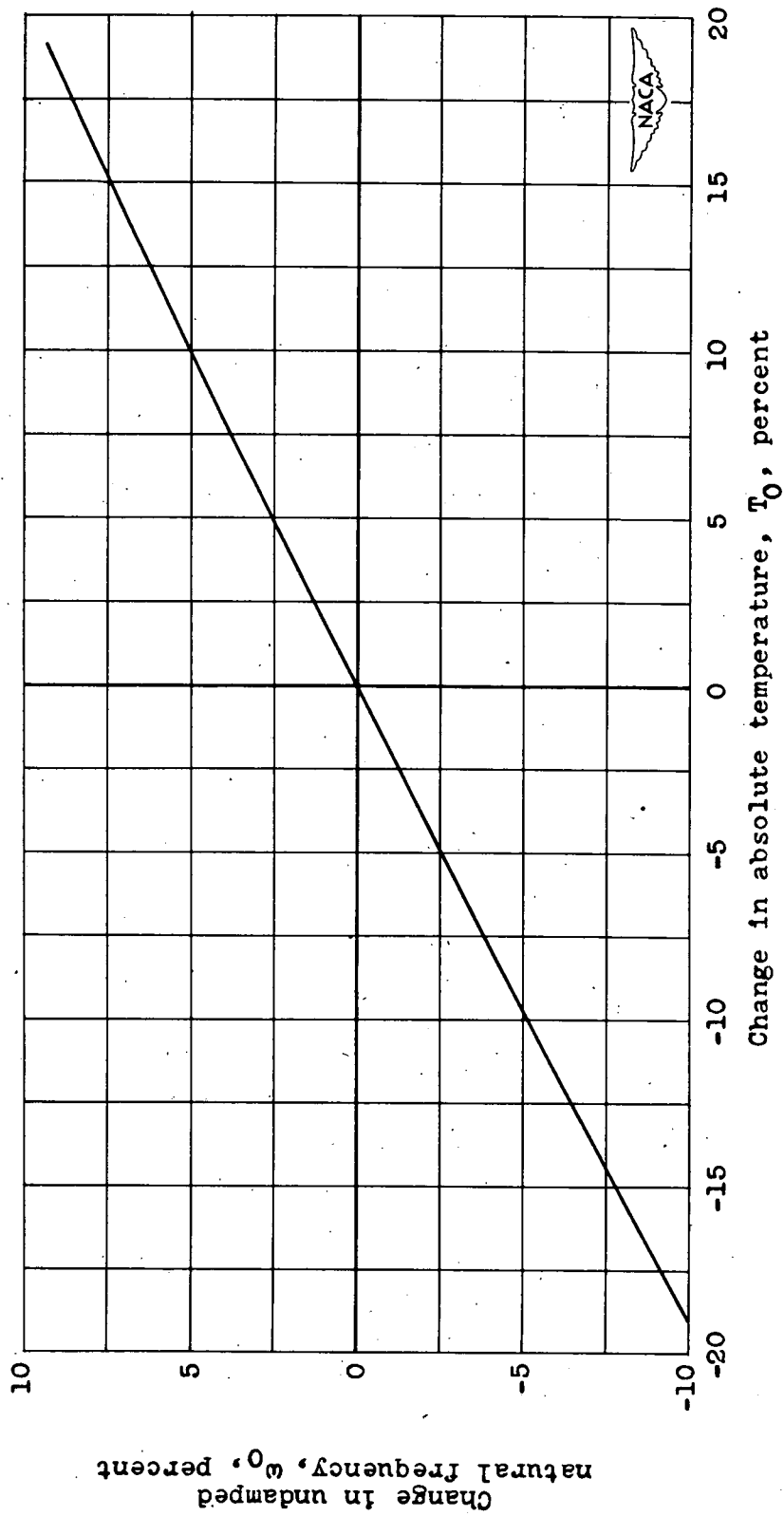


Figure 4. - Variation of undamped natural frequency of gas-pressure-sensing system with operating temperature.

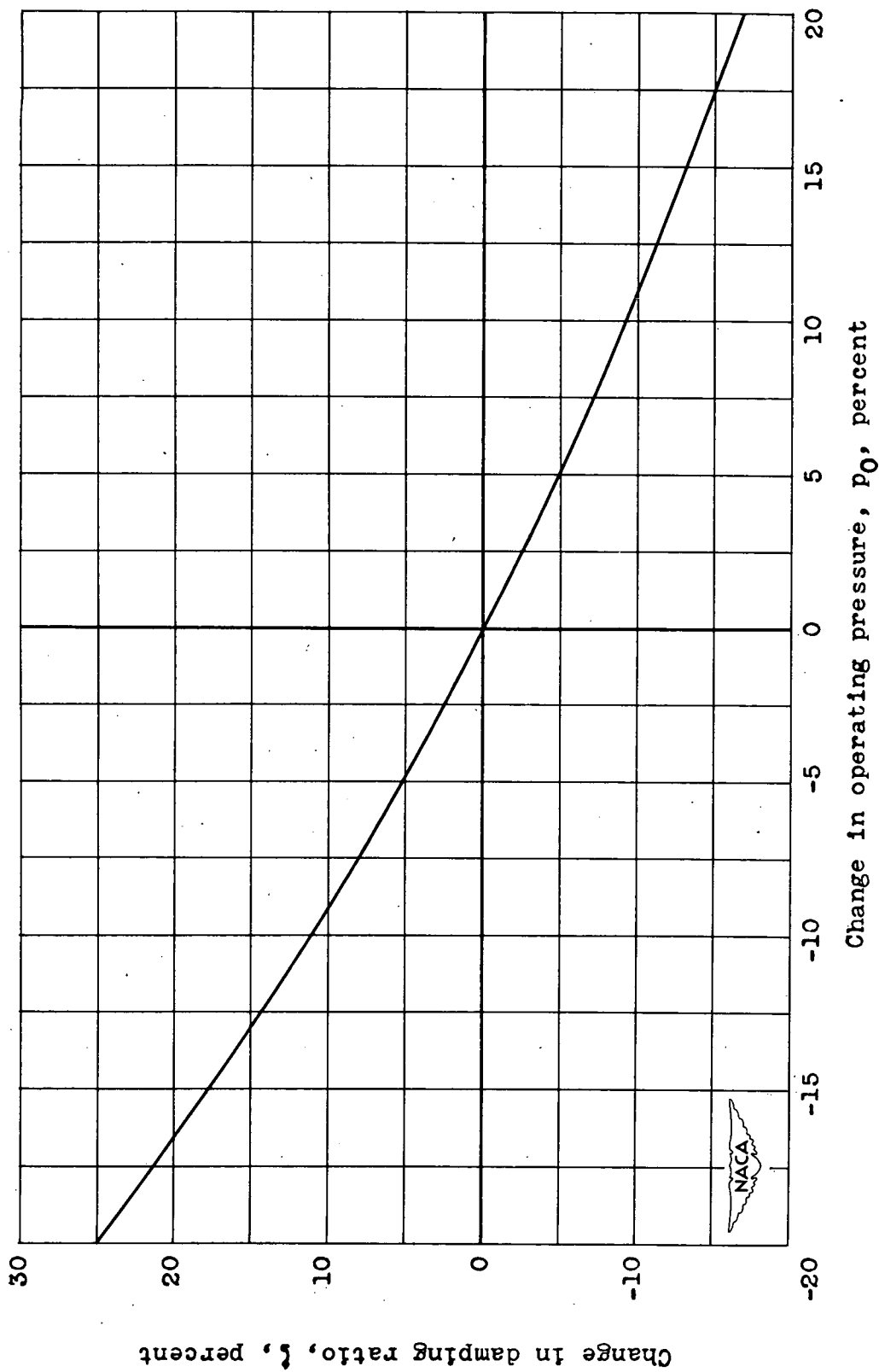


Figure 5. - Variation of damping ratio of pressure-sensing system with change of operating pressure.

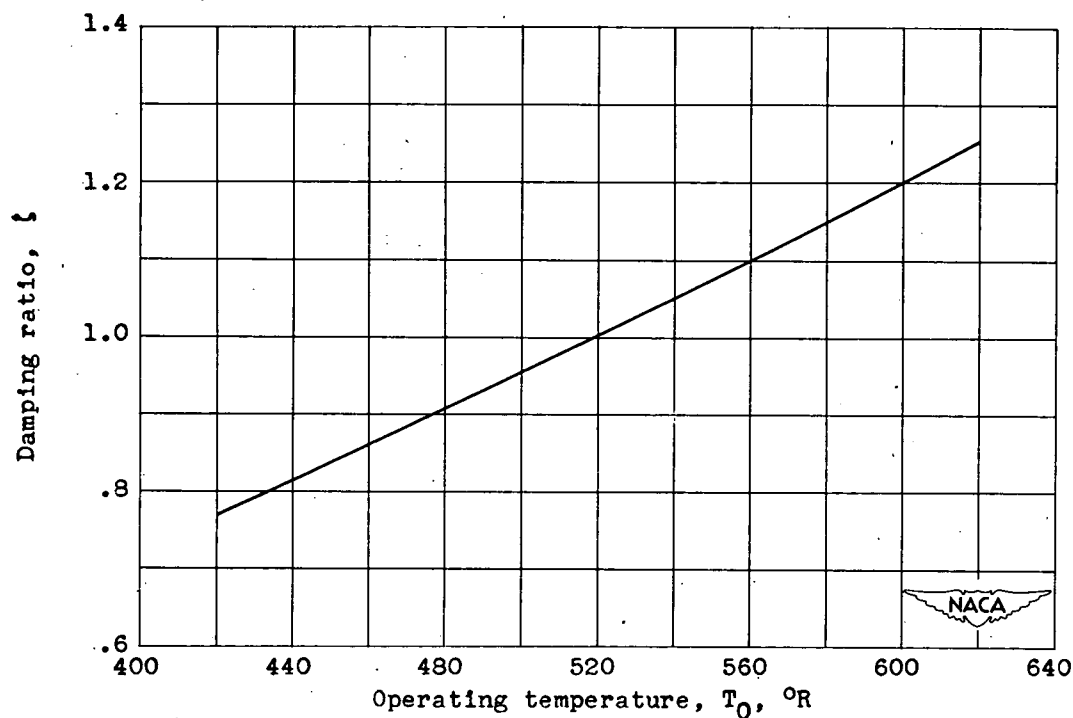


Figure 6. - Variation of damping ratio of pressure-sensing system with operating temperature of various constant values of operating pressure. Operating fluid, air; damping ratio, 1 at standard temperature.

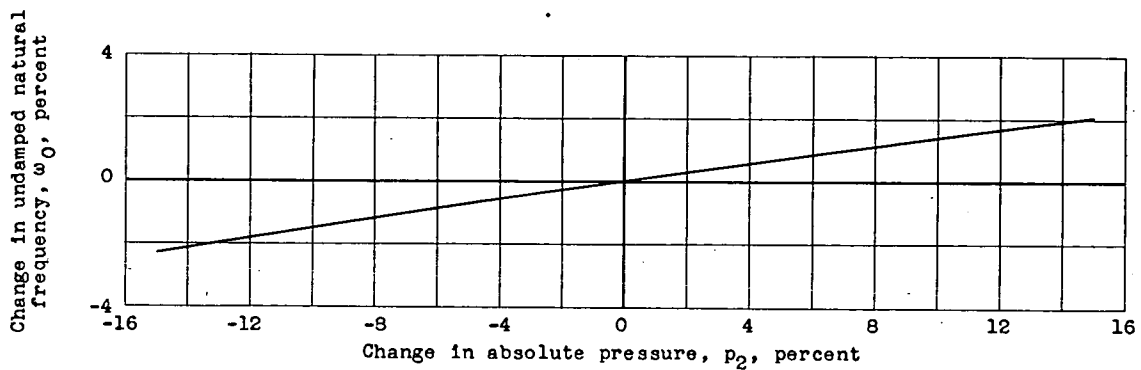


Figure 7. - Variation of undamped natural frequency of pressure-sensing system with change in absolute pressure during pressure transient.

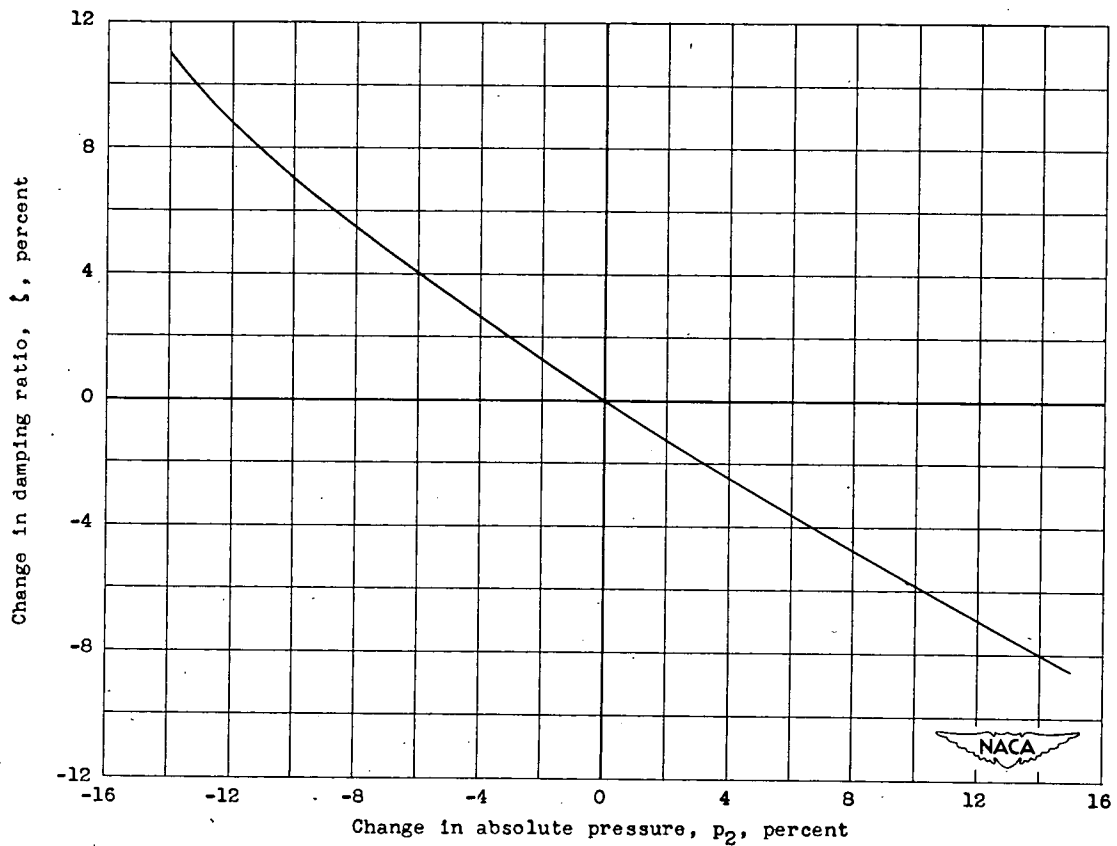


Figure 8. - Variation of damping ratio of pressure-sensing system with change of absolute pressure during pressure transient. Operating fluid, air; initial steady-state absolute temperature, 59° F.

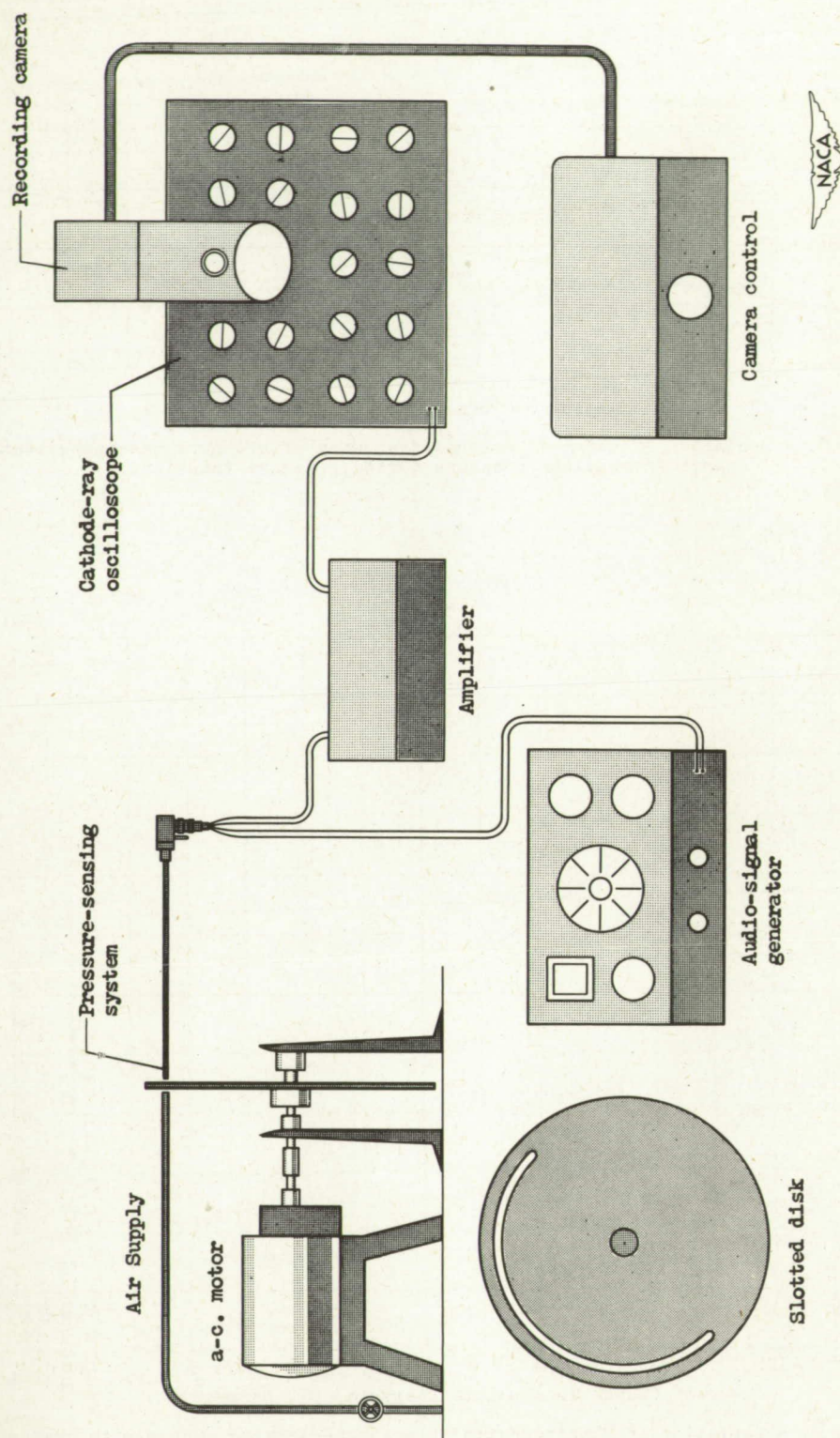
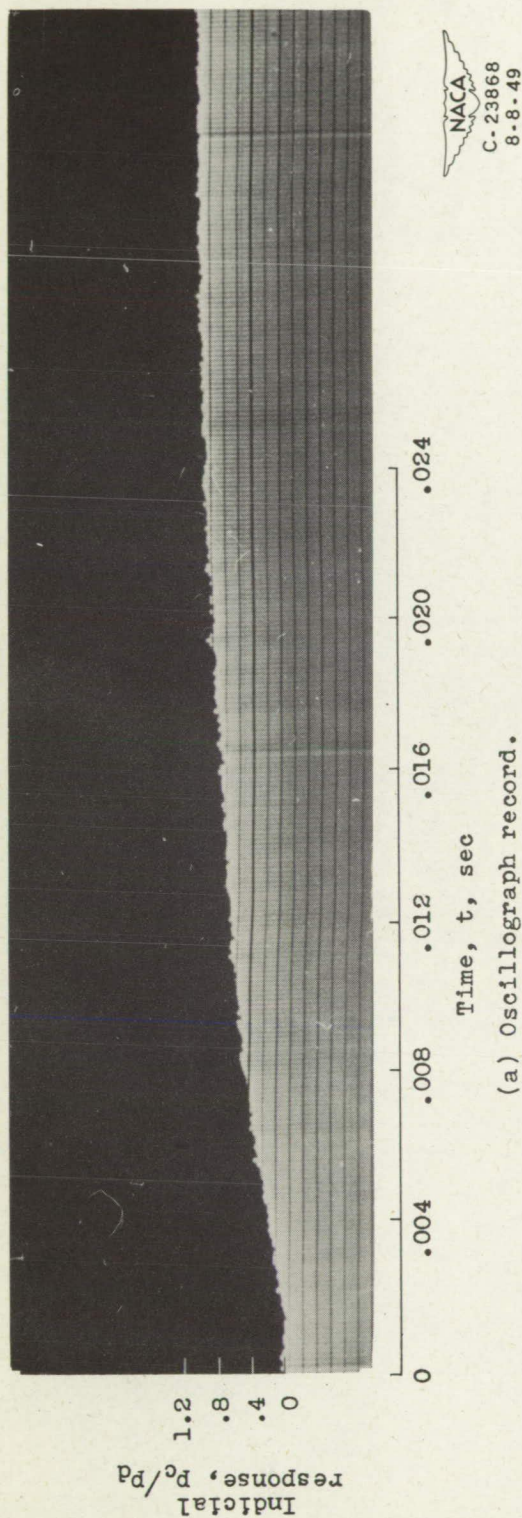


Figure 9. - Experimental apparatus used to obtain transient pressure data.



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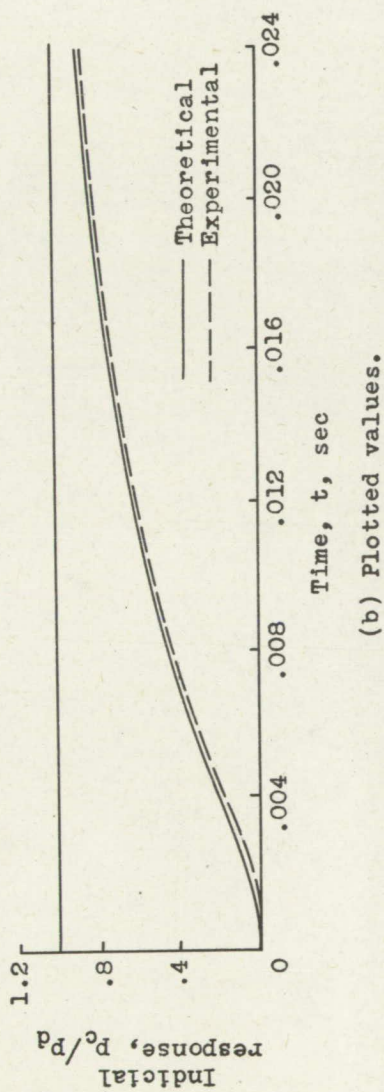


Figure 10. - Indicial response of system with calculated damping ratio of 1.35. Undamped natural frequency, 227 radians per second; initial pressure, 29.32 inches mercury absolute; pressure disturbance, 1.00 inch mercury; initial temperature, 5250 R; tube radius, 0.0175 inch; tube length, 16.75 inches; reservoir volume, 0.202 cubic inch; volume ratio, 0.08.

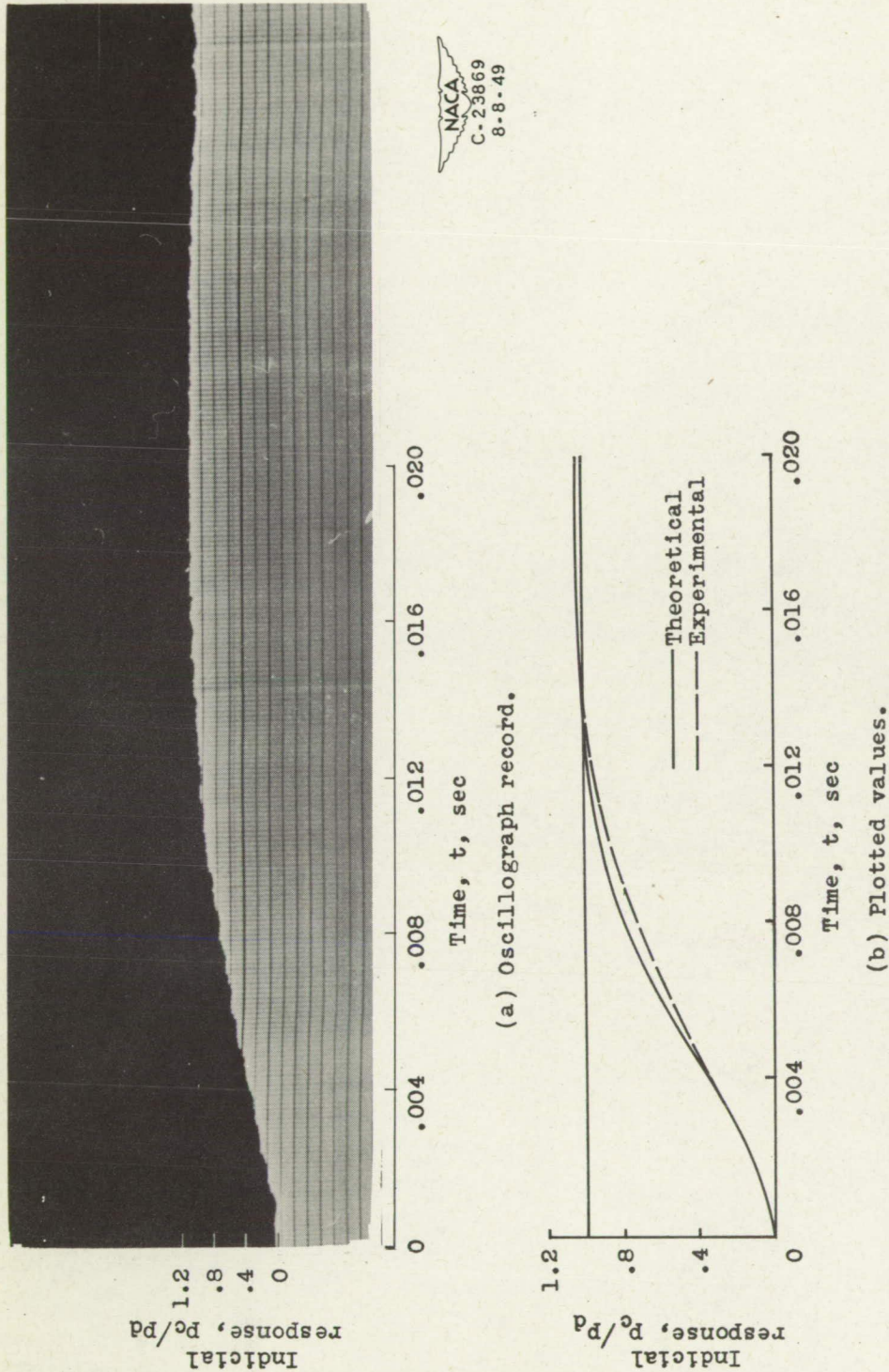
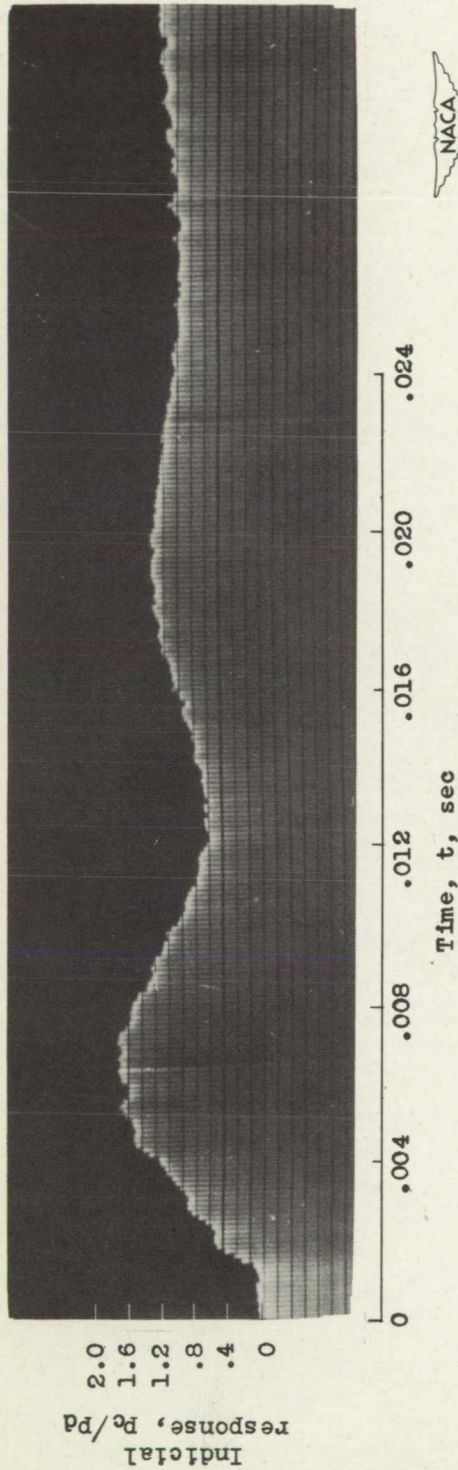
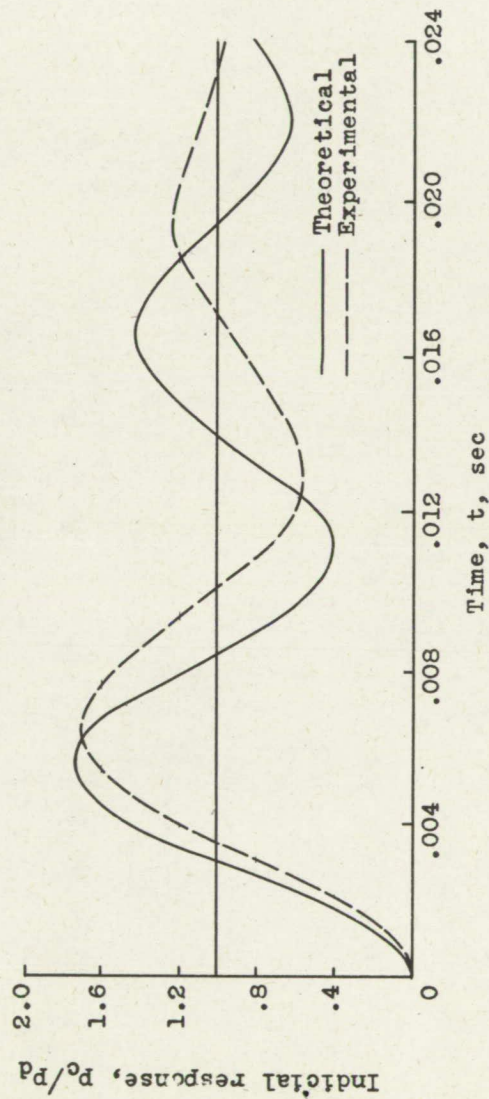


Figure 11. - Initial response of system with calculated damping ratio of 0.727. Undamped natural frequency, 279 radians per second; initial pressure, 29.32 inches mercury absolute; pressure disturbance, 1.00 inch mercury; initial temperature, 5250 R; tube radius, 0.0215 inch; tube length, 16.75 inches; reservoir volume, 0.202 cubic inch; volume ratio, 0.12.



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(a) Oscilloscope record.



(b) Plotted values.

Figure 12. - Indicial response of system with calculated damping ratio of 0.085. Undamped natural frequency, 571 radians per second; initial pressure, 29.32 inches mercury absolute; pressure disturbance, 1.00 inch mercury; initial temperature, 525° R; tube radius, 0.044 inch; tube length, 16.75 inches; reservoir volume, 0.202 cubic inch; volume ratio, 0.50.

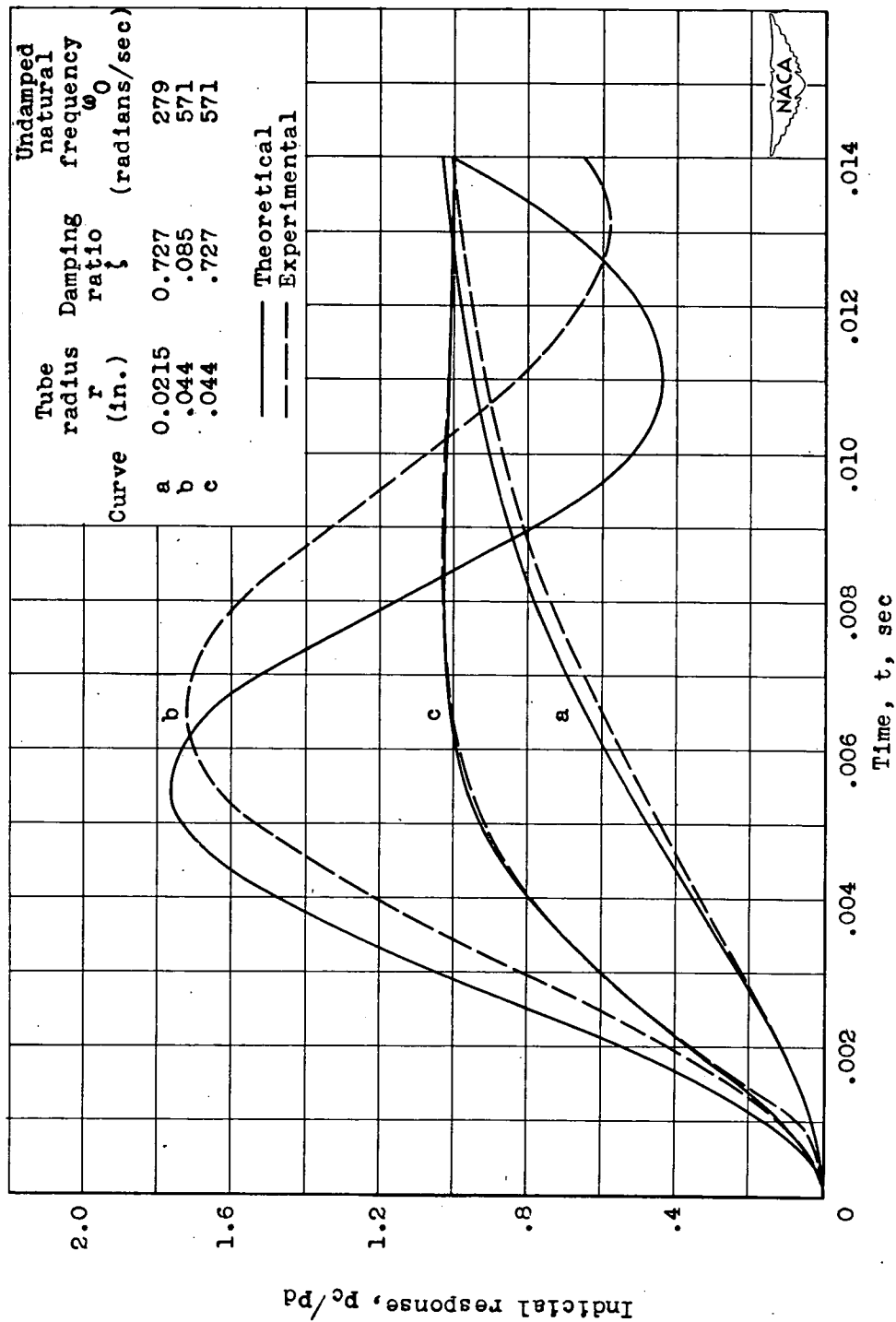


Figure 13. - Theoretical and experimental indicial responses illustrating effect of inserting additional resistance in tube. Initial pressure, 29.32 inches mercury absolute; pressure disturbance, 1.00 inch mercury; initial temperature, 525° R; tube length, 16.75 inches; reservoir volume, 0.202 cubic inch.